Cooperative Mathematics

for level eight

Robin McIntyre

STRAND	MIX AND MATCH	INFORMATION SHARING	SEQUENCING	LANGAUGE MATHEMATICS	EXTRA GROUP ACTIVITIES
Measurement and Calculus	Functions and Gradient Functions P13	Maximising q1-3 Exponential function q4-6 Uncertainties q7,8 Algorithms q9-11	Uncertainties P35	Uncertainties P52 Algorithms P53	
Algebra	Combinations, Permutations and Others P1 Piecewise functions P3 Hyperbolae P4 Algebra P5 Simultaneous Equations with Inconsistent Solutions P6 Linear Programming P7 Curve Fitting P9 Numerical Methods P11 Sequences and Series P22	Sequences and Series q12-14 Hyperbolas q15,16 Piecewise functions q17-19 Binomial Theorem q20,21 Simultaneous Equations q22,23	Remainder and Factor Theorem Proofs P34 Combination Proofs P36 Sigma Proofs P37 Simultaneous Equations P39 Linear Programming P41	Algebra P54 Numerical Methods P53 Sequences and Series P55 Functions P56	
Statistics	Data displays and values P15 Binomial, Normal and Poisson distributions (1) P17, Binomial, normal and Poisson distributions (2) P19, Sample and population statistics P21	Confidence Intervals q24-27 Probability Trees q28-30 Expected Value q31 Binomial Distribution q32,33 Poisson Distribution q34,35	Standard Deviation Proof P37 Expectation Proofs P38 Standard Normal Distribution P43 Confidence Intervals P45	Probability and Statistics P57	Experiment Design P47 Simulations P49

P = PAGE NUMBER q = QUESTION NUMBER

Mathematicians on the front cover are; from top left clockwise - *Hypatia, Pierre-Simon Laplace, Sophie Germain and Blaise Pascal.*

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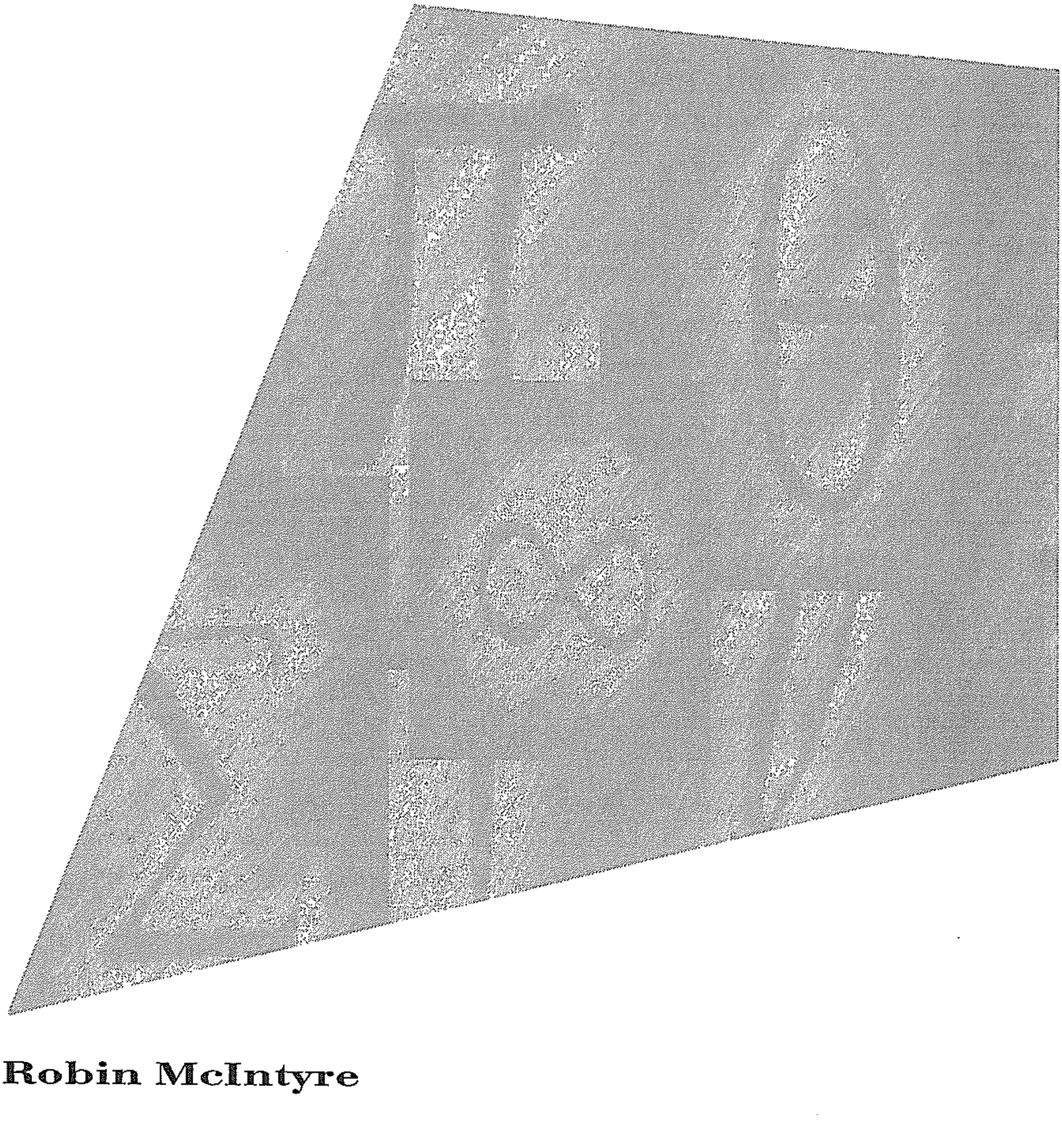

Robin McIntyre

Dedication

To my parents, for everything.

Acknowledgement

My thanks go to the teachers and students who trialled these activities.
Thanks also to Trevor for his continual support and encouragement.

Robin McIntyre
1995

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Contents

Introduction

This book contains activities that are designed to reinforce the mathematical concepts and skills taught in senior mathematics programmes. The activities can be used by groups of students who enhance their learning by discussing and practising the mathematical terminology and skills, while working together in a co-operative and enjoyable way.

There are five styles of activity in this book, each of which can be used in many different ways (see below for some ideas). An explanation on how to use the Language Mathematics Activities is included at the beginning of that section, on pages 51 and 52.

The activities are grouped according to style (see the contents page). The table on the inside of the front cover shows the activities according to topic and style.

It is hoped that students will enjoy doing these activities, and also learn from working together with others. Most activities include students using mathematical processes. For easy teacher reference the content achievement objectives covered or partially covered by the activity are written at the base of each page.

Hints for Teachers

These activities can be set as tasks to be done individually by students. However they were written with groupwork in mind. Groupwork encourages student discussion of the concepts involved. Whether done individually or in groups these activities provide an alternative and enjoyable means of learning level eight mathematics.

The grid shows where to find activities in the book on specific topic areas.

You can use the activities in a variety of ways. Here are a few;
Incorporate them into your normal teaching programme by:

- using an activity or activities as part of a normal lesson.
- using a set of activities from the same topic area as a complete lesson.
- using a collection of activities from different topics as a revision lesson.
- introducing a new topic or skill using an activity as a lesson starter.
- using an activity to revise concepts taught the previous year.

Ideas on how to set up groups

Three students make an ideal group size.

There are many ways to splitting your class into groups. Here are a few:

- straight from the roll in alphabetical order.
- people with birthdays in the same month.
- students with the same number of letters in their first name.
- specific groups chosen by the teacher using some criteria (eg ability, gender balanced, ethnic mix etc).
- draw names from a hat.
- students draw cards out of a hat. The cards each have a mathematical symbol (eg +, -, =, $, % ...). There should be three of each kind of symbol. All the students with matching symbols get together.

Desks may need a quick rearrangement in order to have an ideal classroom set up. Train the class to lift the desks quietly into a group format. Leave room between the groups for the teacher to circulate and have at least three desks for each group so that they have enough space to work effectively. A quiet environment is best for group discussion and student thinking, as well as teacher sanity!!

If you wish you can have each group assign specific jobs to group members, such as recorder, chairperson, enthusiast, etc. These jobs should rotate with each activity so each group member has experience in each position.

Most of these activities have been trialled by schools in Wellington, Auckland and Christchurch.

MMA Ideas for using Mix and Match Activities

- Match each of the graphs, equations and phrases on the left hand page with the numbered statements from the right hand page.
- Write mathematical statements for each of the graphs etc on the left hand page, without looking at the right hand page.
- Make up two more problems with descriptor statements for each. Mix up descriptors and swap with another group.
- Use the problems in conjunction with the answers to help with revision.

SSA Ideas for using Single Sequencing Activities

- Put the steps into the correct order to solve the problem.
- Put the steps into the correct order and either write a sentence to describe the mathematics that has been used at each step or discuss what happened at each step of the problem.
- Make two similar problems, mix up the steps, and swap with another group.
- Use the problems in conjunction with the answers as an aid to revision.

ISA Ideas for using Information Sharing Activities

These problems are split into three sets of clues. Each group member can have one set of clues. Each person shares their clues with the other group members, in order to compile enough information to solve the problems.

- Exchange information by talking only, so that each group member can draw the graph or solve the problem.
- Exchange information by talking only and solve the problem together on one piece of paper.
- Make similar problems to swap with other groups.

DSA Ideas for using Double Sequencing Activities

Put the written descriptions (A-G) into the correct order. Match each with the symbolic part of the solution. These boxes are numbered.

- Find one correct sequence from the answers. Use this to sequence the other part(s) of the activity.
- Use both parts of the answer to learn how to solve problems of that type. Complete some other questions from a text book in the same way.
- Use the sequence of written instructions to solve other problems.

Combinations, Permutations and Others

1. Match each of A,B and C with one of a - c and four numbered statements.
2. Make up one question of each type and swap with another group.

A.

Permutations

B.

Combinations

C.

Multiplication Principle

a.

$$\frac{n!}{(n-r)!r!}$$

b.

$$n_1 x n_2 x n_3 \ldots$$

c.

$$\frac{n!}{(n-r)!}$$

1. Alan Jones takes two of his four children on a Carribean holiday. How many different selections of the two lucky children are possible?

2. How many ways are there of ordering ten books onto a shelf?

3. One student is to be chosen from each of three classes. If there are twenty-five students in each class, how many possible selections are there?

4. Mrs Gracie has twenty-three cats. She decides to take four to the national cat show. How many different selections are possible?

5. In a competition entrants have to rank six cricket catches in order of difficulty. What is the minimum number of different entries you need to make to ensure one of your entries is correct?

6. A restaurant offers three different soups, five entrees, six mains and four desserts. How many different four-course meals are possible?

7. In how many ways can four boys and five girls sit in a row if all the boys must sit together?

8. A committee of four is to be formed from a group of six women and eight men. If the committee has only one man, how many different committees are possible?

Time Savers

- **Each group chooses one of A,B or C to complete.**
- **Groups match 1-12 with A,B and C but solve only one of each kind.**

9. A young hairdresser can do four different styles of perm and can dye hair to any of twelve different shades. How many different hairstyles, which include both a colour and a perm, can he create altogether?

10. Six books are randomly selected from a shelf of 20 books. How many different selections are possible?

11. Five-digit numbers are made by selecting digits from 1,2,3,4,5,6,7,8,9. How many different numbers are possible if the digits may be repeated?

12. Five-digit numbers are made by selecting digits from 1,2,3,4,5,6,7,8,9. How many different numbers are possible if the number starts with 5 and has no digits repeated?

Algebra level 8: ... investigate and find numbers of arrangements and selections from a number of objects.

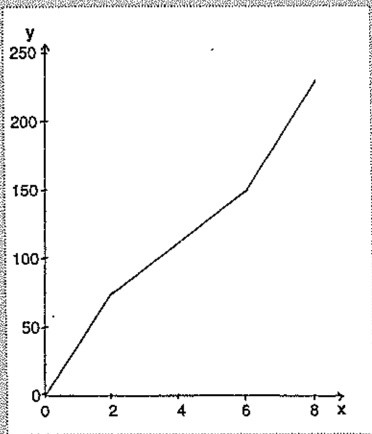
Piecewise Functions

1. Match four numbered statements with each graph.

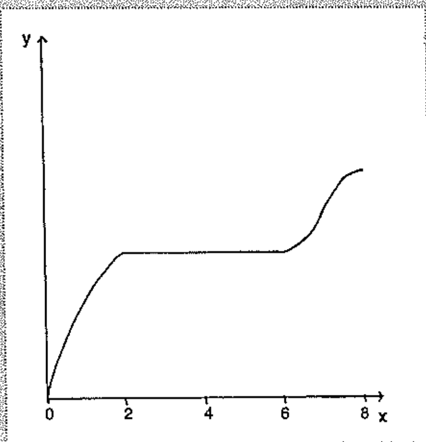
2. Discuss how well each graph models the practical situation you have matched it with.

Decide on which variable is on each axis. Choose appropriate units and decide whether the given values are realistic.

A.



B.



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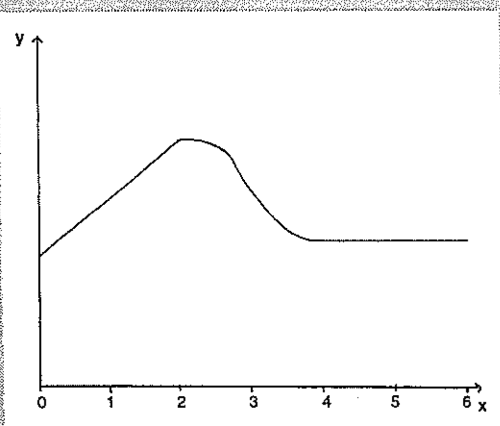


1. $\frac{dy}{dx} \geq 0$ for all x

2. $y = x + 37,\ 0 \leq x \leq 2$

3. A knitter knits plain rows more quickly than purl rows. This graph shows how the number of stitches varies with time for three rows of knitting (rows alternate, purl, plain)

4. $f'(4) < f'(1)$

5. The domain is in three sections.
$$\begin{cases} 0 \leq x \leq 2 \\ 2 < x \leq 4 \\ x > 4 \end{cases}$$

6. This graph shows how the volume of milkshake consumed by a boy at McDonalds varies over time.

7. This graph is of a sick child's temperature one evening, before and after the medicine was given.

8. There is one linear section of this function.

9. All three sections of the function show constant rates.

10. $y = 18.75x + 37.5,\ 2 \leq x \leq 6$

11. $y = 200,\ 2 \leq x \leq 6$

12. The second section of the function is decreasing.

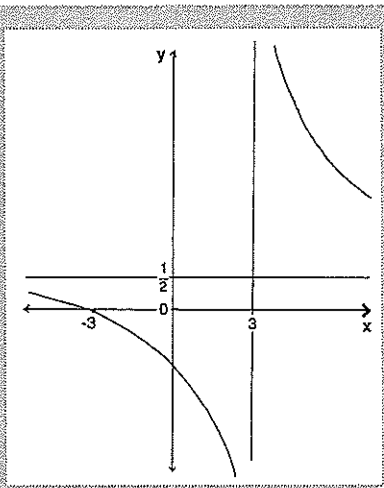
Hyperbolas

1. Match four numbered statements with each graph.

2. Sketch $y = \frac{2x-3}{x+1}$. Write four descriptors for the function.

3. Investigate $y = \frac{ax^2+b}{cx+d}$.

A.



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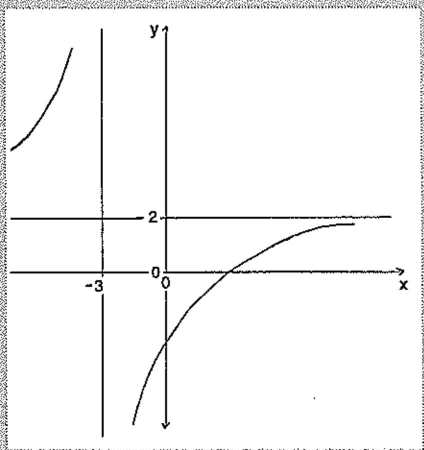
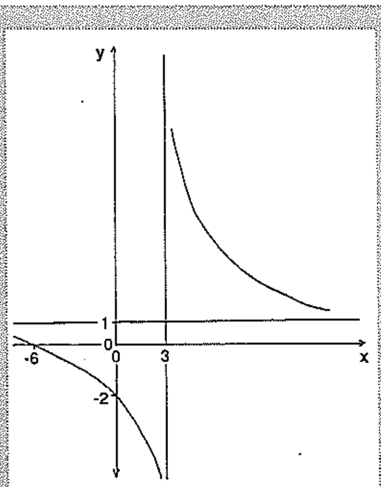


C.



1. As $x \to \infty,\ y \to \frac{1}{2}$

2. The point (–1, –4) is on this function.

3. The y– intercept is the point (0, –2)

4. The curve cuts the x–axis at (–6, 0)

5. The function is $y = \frac{x+3}{2x-6}$

6. The function is all the points where $y = \frac{x+6}{x-3}$

7. The x– intercept is the point (–3, 0)

8. As $x = -\infty,\ y \to 1$

9. The curve is defined by $y = \frac{2x-6}{x+3}$

10. As $y \to -\infty,\ x \to 3$

11. The horizontal asymptote is the line y=1.

12. The curve is $y = 2 - \frac{12}{x+3}$

Algebra level 8: ... Use graphical techniques to explore and illustrate** $y = x^a, y = \frac{ax+b}{cx+d}$**, and piecewise functions.

MMA Algebra

1. Match four numbered boxes with each of A,B,C and D.
2. Answer all the numbered boxes.
3. Find or write two more problems of each type, and answer them, or swap with another group.
4. Answer the "Algebra in Context" problems from the bottom box.

Time Savers

1. Each group finds one numbered problem of each type and solves it.
2. Each group member finds and solves a different set of problems A - D.

A. Factorise

B. Simplify

C. Change the subject of the formula

D. Solve

Algebra in context:

1. The area of a regular hexagon is given by the formula $A=\frac{1}{2}aP$ where P is the perimeter of the hexagon and a is as shown on the diagram.

(i) Find the area of a hexagon with $a=\sqrt{3}$m and side length 2m.

(ii) Find a if the area is 400 cm² and the perimeter is 45cm.

(iii) Prove $A=\frac{1}{2}aP$ for all regular polygons.

2. The formula $v=d+drt$ represents the value of an investment of d dollars at a rate of interest, r, for t years.

(i) Make r the subject of the formula.

(ii) Calculate v for a \$2000 investment made for 2 years at 7% interest.

(iii) Make t the subject and state why this version of the formula might be useful.

1. $A=\frac{1}{2}r^2\theta$ (θ)	**9.** $\frac{\sqrt{x}}{7}=53$
2. $\frac{b^n x b^{n+3}}{b^4}$	**10.** $x^2+3x-18$
3. $S=\frac{n}{2}(a+\ell)$ (a)	**11.** $6km-9kt-2gm+3gt$
4. $x^2-9x=10$	**12.** $\frac{1}{3x}-\frac{1}{2x}=5$
5. $\frac{1}{x^2}=\frac{1}{a}-\frac{1}{b}$ (a)	**13.** $T=2\pi\sqrt{\frac{\ell}{g}}$ (ℓ)
6. $4t^2-9$	**14.** $\sqrt[3]{\frac{27x}{8y^3}}$
7. $2x(x+3)-y(x+3)$	**15.** $(4x-3)^2-2x=11$
8. $\frac{1}{x}\cdot\frac{x^2+2x}{y}$	**16.** $3\log x+2\log y$

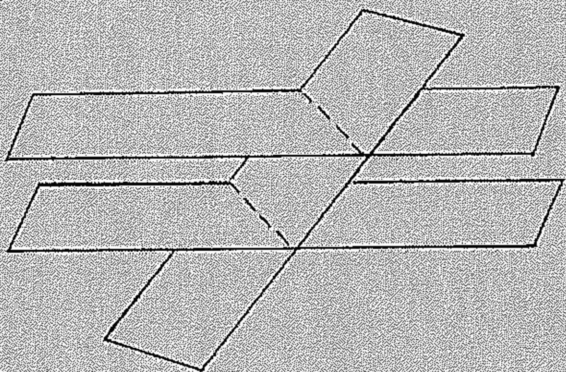
Simultaneous Equations with Inconsistent Solutions

1. Find the four numbered statements that match each of A,B and C. Justify your choices.

2. Make up one set of simultaneous equations for each type of inconsistent solution.

A.

B.



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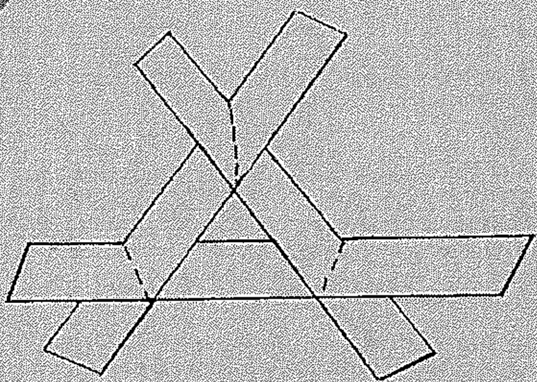


1. $x + 7y + 5z = 10$
$x + 5y + 4z = 5$
$x + 3y + 3z = 4$

2. Two of the planes are parallel.

3. $x + y + z = 5$
$3x + 4y + 2z = 11$
$3x + 4y + 2z = 13$

4. The lines of intersection of pairs of planes are parallel.

5. $2x + 3y + z = 8$
$2x + 3y + z = 12$
$6x + 9y + 3z = 3$

6. Each plane intersects two other planes.

7. The three planes are parallel.

8. One plane intersects with two other planes.

9. $3x + 5y + 3z = 12$
$2x + 3y + 3z = 3$
$x + y + 3z = 0$

10. $x + y + z = 11$
$2x + 2y + 2z = 13$
$2x + 2y + 2z = 23$

11. One plane intersects neither of the other planes.

12. $2x + 2y + z = 1$
$4x + 4y + 2z = 3$
$2x + 4y + 2z = 9$

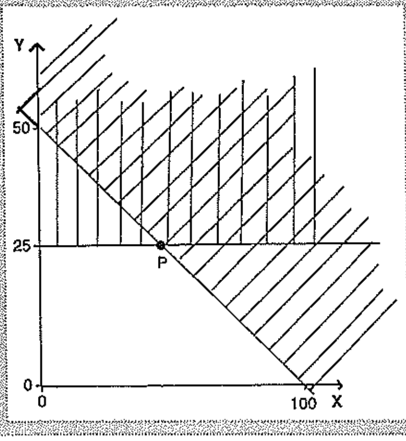
Algebra level 8:... use simultaneous equations to model real and simulated solutions and interpret their results in a given context.

Linear Programming

The required region is left unshaded. In all cases $x \geq 0$ and $y \geq 0$.

1. **Match four statements with each graph.**
2. **Draw a set of axes and mark on it three inequality regions. Write an appropriate algebraic inequality statement for each region you have shaded.**

A.



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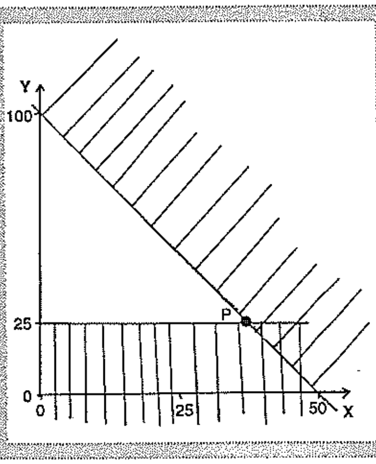

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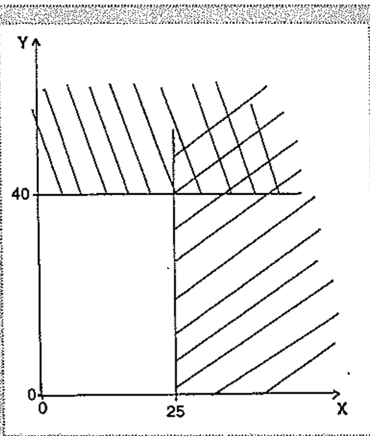
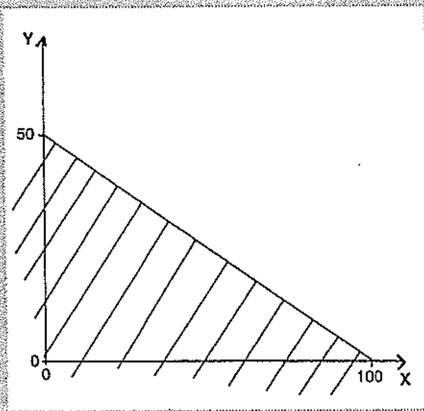
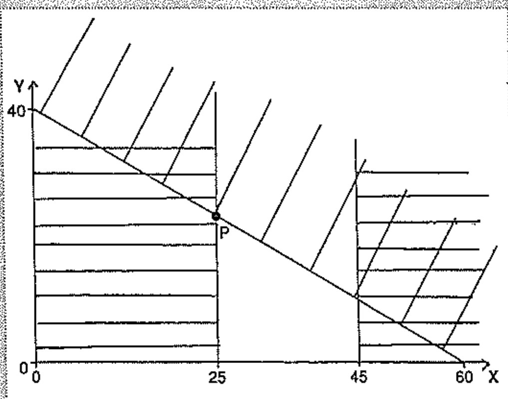


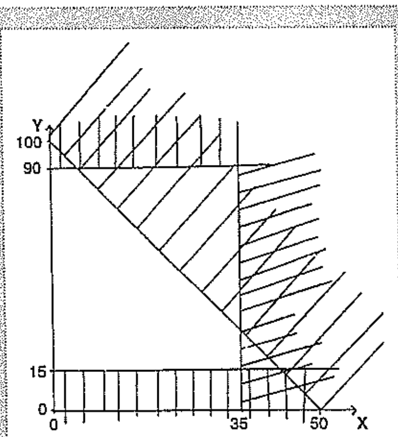
D.



E.



F.



1. $x \geq 25$

2. $2x + 3y \leq 120$

3. The solution set is triangular.

4. P is the point $(25, 23\frac{1}{3})$.

5. $x + 2y \leq 100$

6. $2x + y \leq 100$

7. (25,40) is a possible solution.

8. $y \leq 40$

9. The solution set is infinite.

10. $25 \leq x \leq 45$

11. (80,0) is a possible solution.

12. $15 \leq y \leq 90$

13. $x + 2y \geq 100$

14. $2x + y < 100$

15. $y \geq 25$

16. $x \leq 25$

Time Savers

- Each group finds the four correct statements for only one or two of the graphs.
- Each group writes four statements for one or more of the graphs.

17. Profit = $30x + 500y$. The maximum profit is \$20,750.

18. There is one inequality shown on the graph.

19. $y \leq 90$

20. P is the point (50,25)

21. Cost = $60x + 27y$. The minimum cost is \$1350.

22. There are four constraints on x and y.

23. Volume = $3x + 4.5y$.
The maximum volume is given by the solution (100,0).

24. P is the point (37.5, 25).

Algebra level 8:...model real simulated situations, using linear programming techniques, and obtain and interpret optimal solutions.

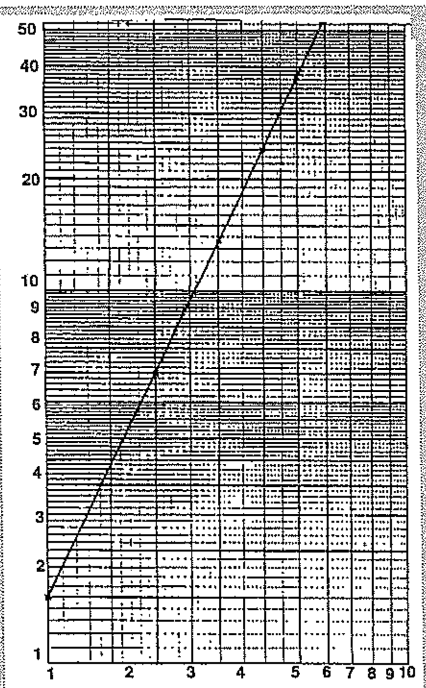
Curve Fitting (Linear, Log/Linear and Log/Log)

1. Match four numbered statements to each graph labelled A,B and C.
2. On your own paper sketch the three functions 1,2, and 12. Describe for each a real life situation that could generate the function. eg. the distance travelled by a girl walking her dog, the oven temperature as it heats up, how hungry you are between breakfast and lunch, the volume of windscreen fluid with the number of squirts used, the population of bacteria on a sandwich left in a locker over time, etc.
3. For each set of data (a) - (c), (i) plot a graph of x and y,

   (ii) use a linear, log-log or log-linear graph to find the function that links x and y.

   (iii) discuss whether or not the data is realistic.

A.



1. $y = \frac{x}{15} + 1$

2. $y = 15e^{0.2x}$

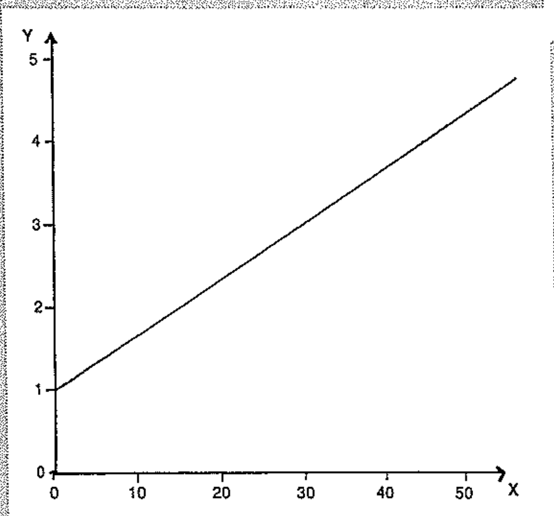
3. The y-intercept of this function is (0,15).

4. This graph shows that x and y are related by a power law.

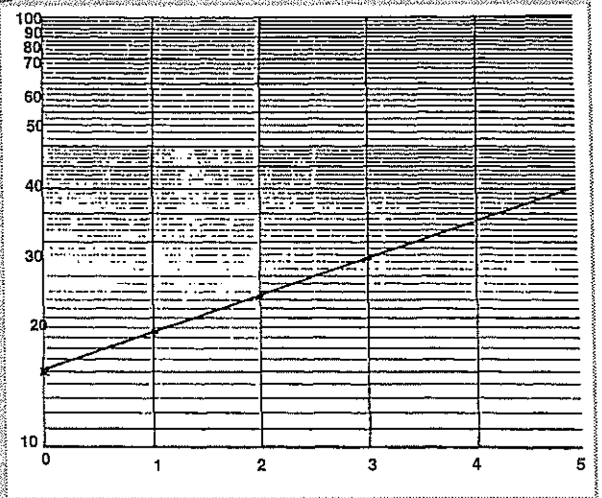
5. Either $\log_{10}$ or ln can be used to find the equation.

6. If the points on this graph were not in a straight line, the function would not be an exponential one.

B.



C.



7. This graph shows a linear relationship between x and y.

8. The gradient of this line is 2.

9. This graph shows that there is an exponential relationship between x and y.

10. The gradient of this function is $0.0\dot{6}$.

11. This function has a constant rate of change.

12. $y = 1.5x^2$

Time Savers

1. Each group chooses one of A,B OR C to match.
2. Each group chooses one equation, 1, 2, or 12 to match.
3. Each group chooses one set of data (a), (b) or (c) to match.

(a) x= age of a baby in months.
y= length of the baby (cm).

x	0	1	2	3	4	5	6	7	8
y	50	50.9	51.8	52.7	53.6	54.5	55.4	56.3	57.2

(b) x= the number of months since January 1995.
y= the number of possums on a farm.

x	0	2	4	6	8	10	12	14
y	16	19	24	29	36	43	53	65

(c) x= distance in metres from the shore of a beach.
y= depth of the water in metres.

x	0	10	20	30	40	50	60	70
y	0	11.7	16.5	20.3	23.4	26.2	28.7	31

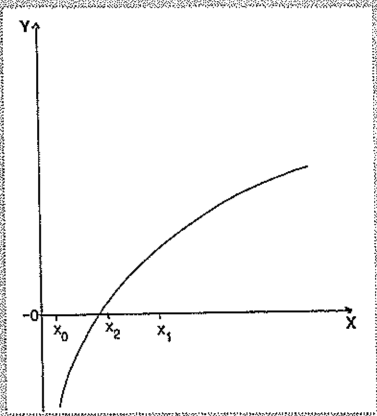
Algebra level 8:...choose an appropriate model for real data, including the use of a Log-Log and Semi-Log techniques, and analyse and interpret the results.

Numerical Methods for Solving Equations:

1. **Match each graph with one table and four numbered statements.**
2. **Continue each table until four iterations are complete.**

   The function is $y = 3x - e^x$.
3. **Compare the tables. You should comment on : how easy the calculations were, any similarities between the tables, any advantages of one method over the others......**

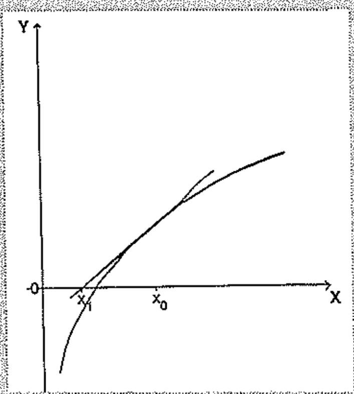
A.



a.

x_0	y_0	x_1	y_1
0.5	−0.1487	1.0	0.2817
1.0	0.2817	0.6727	

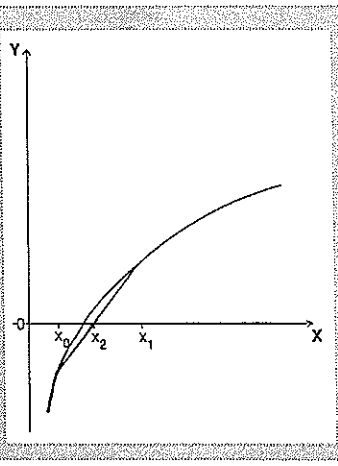
B.



b.

x_1	y_1	y_1'
0.5	−0.1487	1.3513
0.6100	−0.0104	

C.



c.

x_0	y_0	x_1	y_1	x_2	y_2
0.5	−0.1487	1.0	0.2817	0.75	0.1330
0.5	−0.1487	0.75	0.1330	0..625	

1. This method uses one starting or initial value.

2. $x_2 = \frac{x_0 + x_1}{2}$

3. The new approximation is the value where the chord cuts the x-axis.

4. $x_2 = \frac{x_0y_1 - x_1y_0}{y_1 - y_0}$

5. This graph shows the Newton-Raphson method.

6. This graph shows the bisection method.

7. This method is slower to converge on the root than other methods.

8. This method will always find the root to the required accuracy.

9. The new approximation is found where the tangent cuts the x-axis.

10. $x_2 = x_1 - \frac{y_1}{y_1'}$

11. The Secant method is shown in the graph.

12. This method requires two starting values.

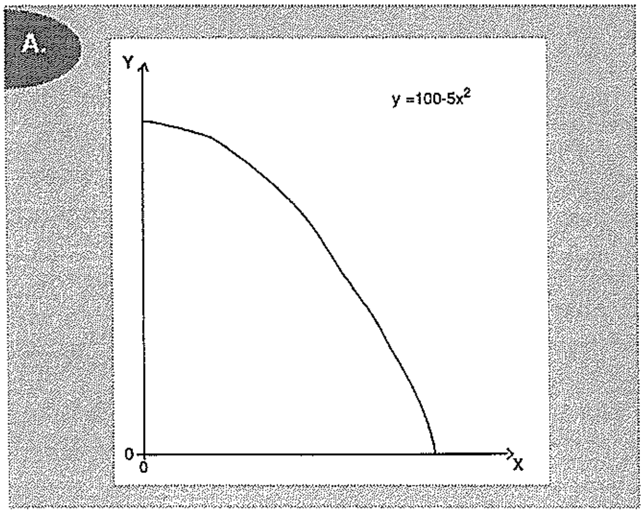
Time Savers

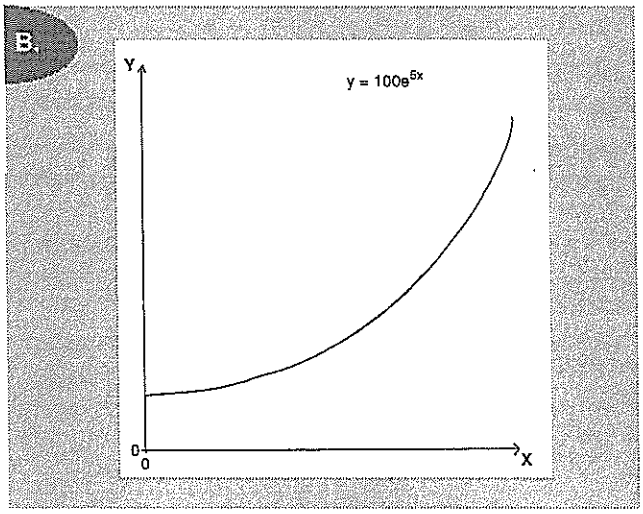
- **Each group matches a table and statements for one or two graphs only.**
- **Each group matches the statements only to the graphs.**
- **Each group matches the tables to the statements.**

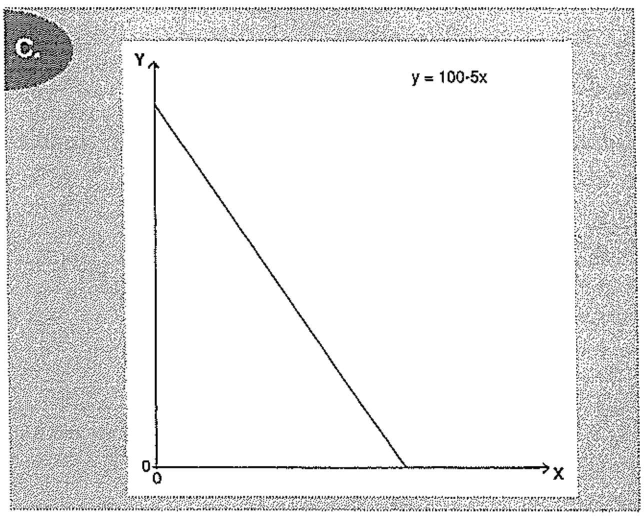
Algebra level 8:...use appropriate numerical methods and technology to solve non-linear equations.

Functions and Gradient Functions

1. Match four numbered statements to each of A,B and C.
2. Match the gradient function graphs to the original graphs.
3. Make up a graph that could describe a practical situation. Write four statements about it and sketch the gradient function.
4. Answer the problems on page 14.







1. This graph may represent the height of a falling ball as time passes.

2. $y' = -10x$

3. The value of the function is zero when x is 20.

4. This graph may represent the population growth for a bacterial population.

5. $y' = 500e^{5x}$

6. The y-intercept for the function is (0, 100).

7. The curve becomes steeper as x increases, y' is always negative.

8. The function has a real root at $x = \pm\sqrt{20}$

9. $y' = -5$

10. The rate of change of the function is constant.

11. The rate of change of the function at a point is proportional to the value of the function at that point.

12. As x increases y decreases.